The Tadpole Conjecture in the Strict Asymptotic Regime

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Based on [arXiv:2204.05331]
with M. Graña, T. Grimm, D. van de Heisteeg, E. Plauschinn

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1. F-theory con Calabi-Yau four-folds and the Tadpole Conjecture
2. Asymptotic Hodge Theory ⟶ Strict Asymptotic Limit and Key Results
3. Moduli stabilization ⟶ General Results and Tadpole Scaling
4. Outlook & Open Questions

F-theory Compactifications

- Consider F-theroy on a Calabi-Yau fourfold with fluxes

Review: [Denef '08]
Effective action:
[Grimm '10]
[Haack, Louis '21]

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- Restrict to complex structure sector: $J \wedge G_4 = 0$

[Becker, Becker '96]
[Dasgupta, Rajesh, Sethi '99]

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- Hodge decomposition and Hodge star: $H^4_{\text{prim}} = H^{4,0} \oplus H^{3,1} \oplus H^{2,2}_{\text{prim}} \oplus H^{1,3} \oplus H^{0,4}$

$$\star\, v^{p,q} = i^{p-q}\, v^{p,q}$$

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Tadpole Conjecture: The flux contribution to the tadpole needed to stabilize a large number of moduli grows as

$$Q > \alpha\, n_{\mathrm{stab}} \qquad \alpha > 1/3 \qquad \frac{\chi(Y_4)}{24} \sim \frac{1}{4} h^{3,1}$$

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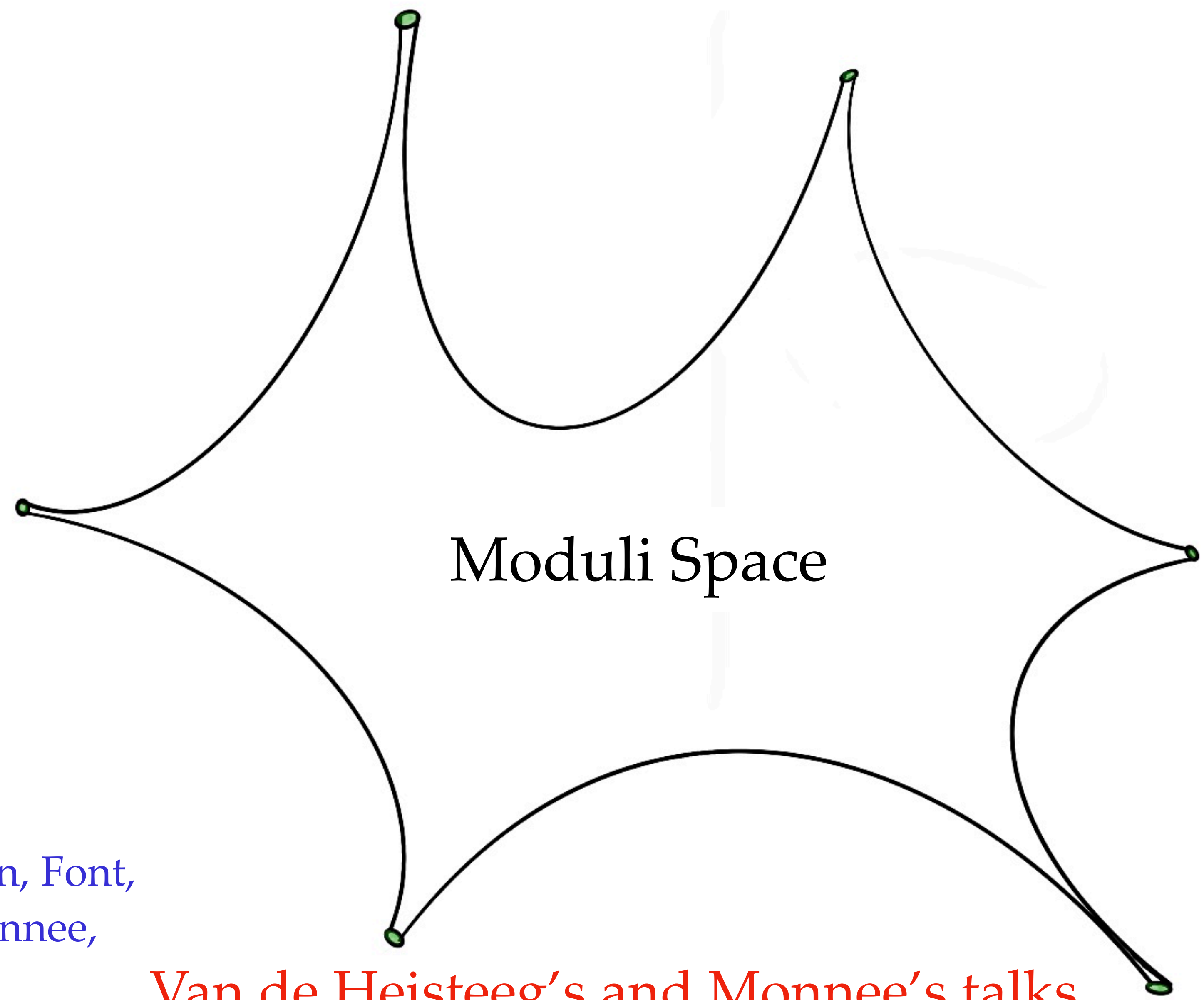
GOAL: Prove this in the strict asymptotic region of moduli space

Asymptotic Hodge Theory
-Asymptotic limits-

[Griffiths, Deligne, Schmid, Cattani, Kaplan…]



[Grimm, Palti, Valenzuela, Li, Bastian, Castellano, Calderón-Infante, Corvilain, Font, Gendler, Van de Heisteeg, Ibáñez, Monnee, Plauschinn, Uranga…]

Van de Heisteeg's and Monnee's talks

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  $t^i = \phi^i + is^i$
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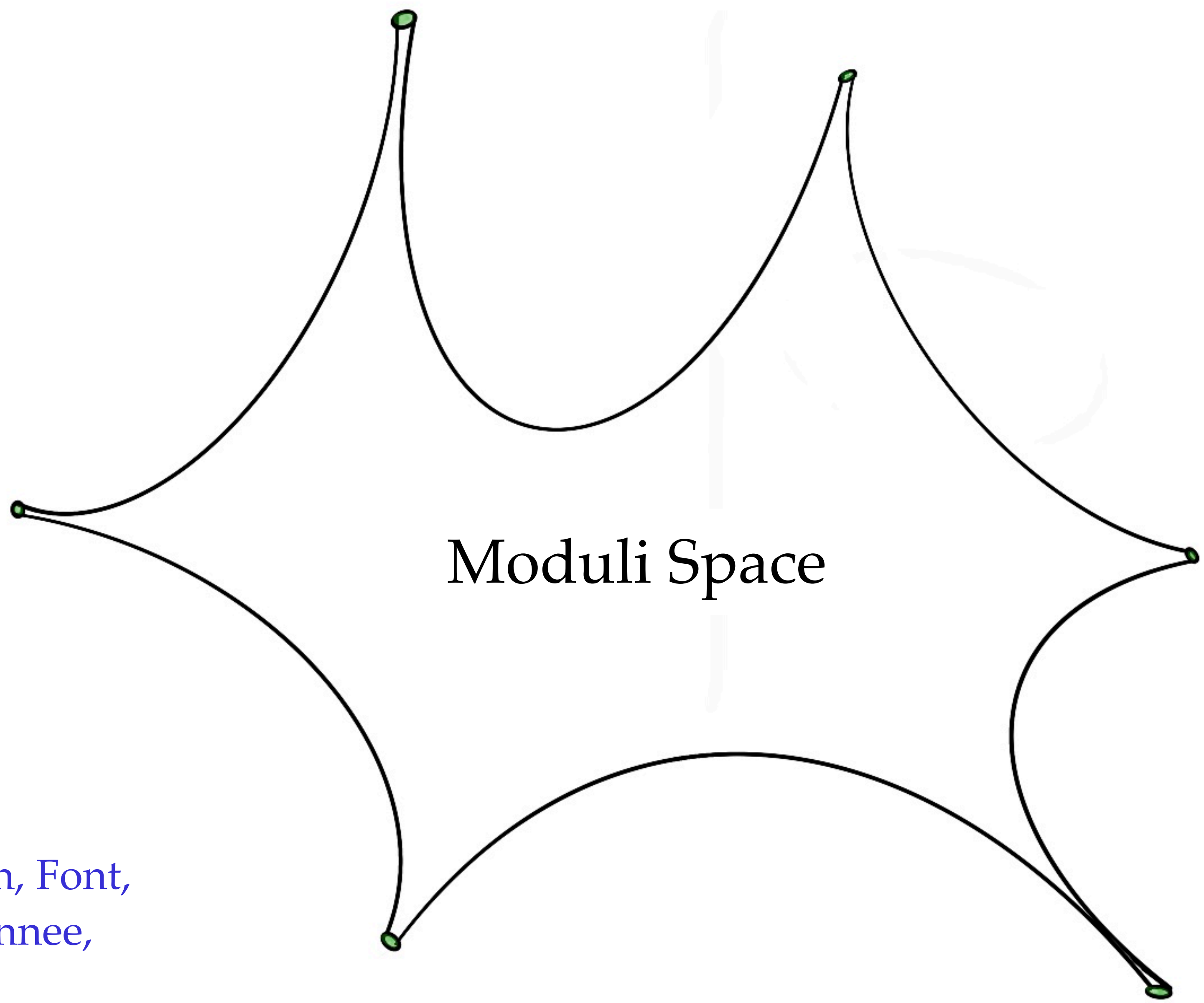

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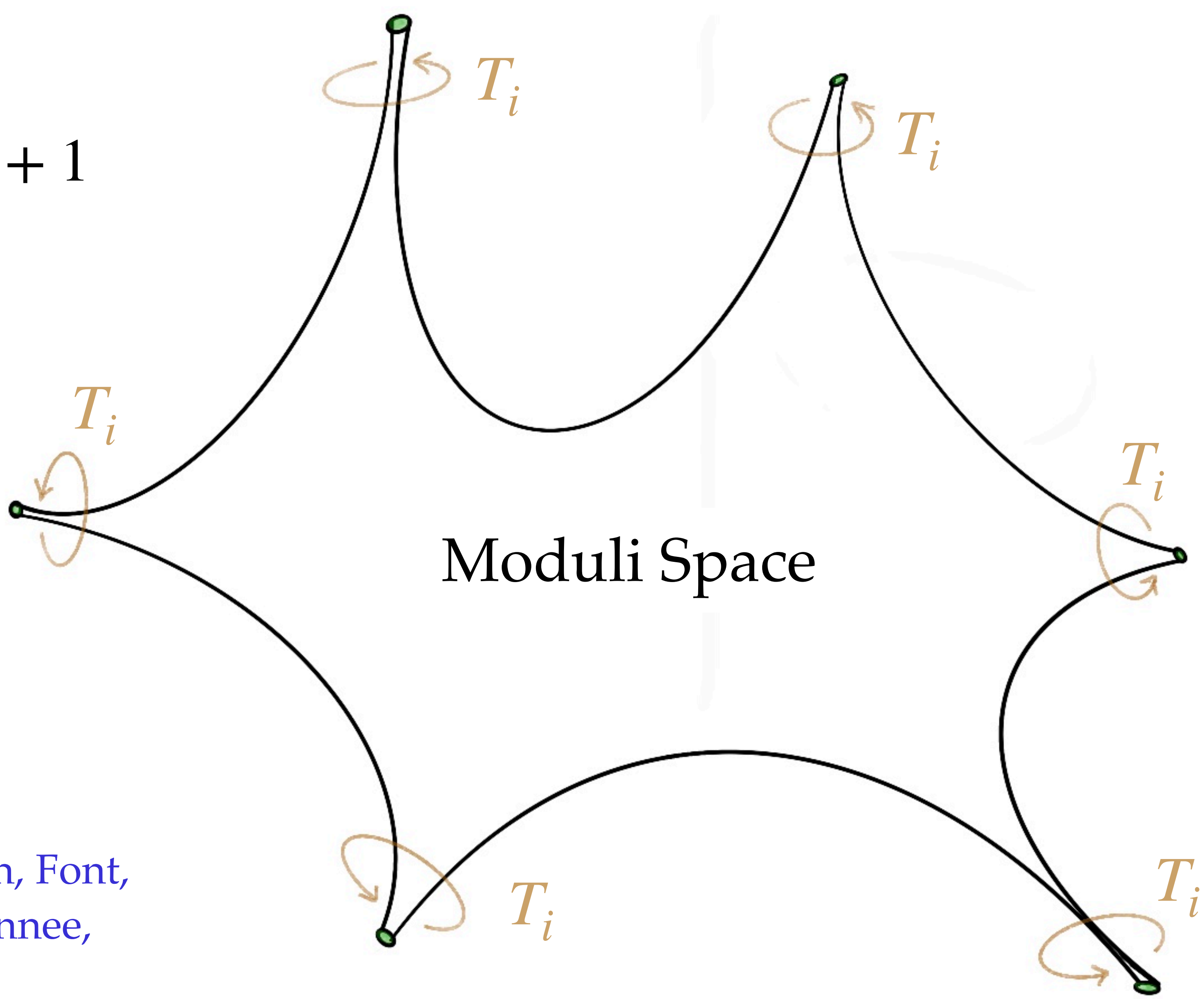
$T_i = e^{N_i}$



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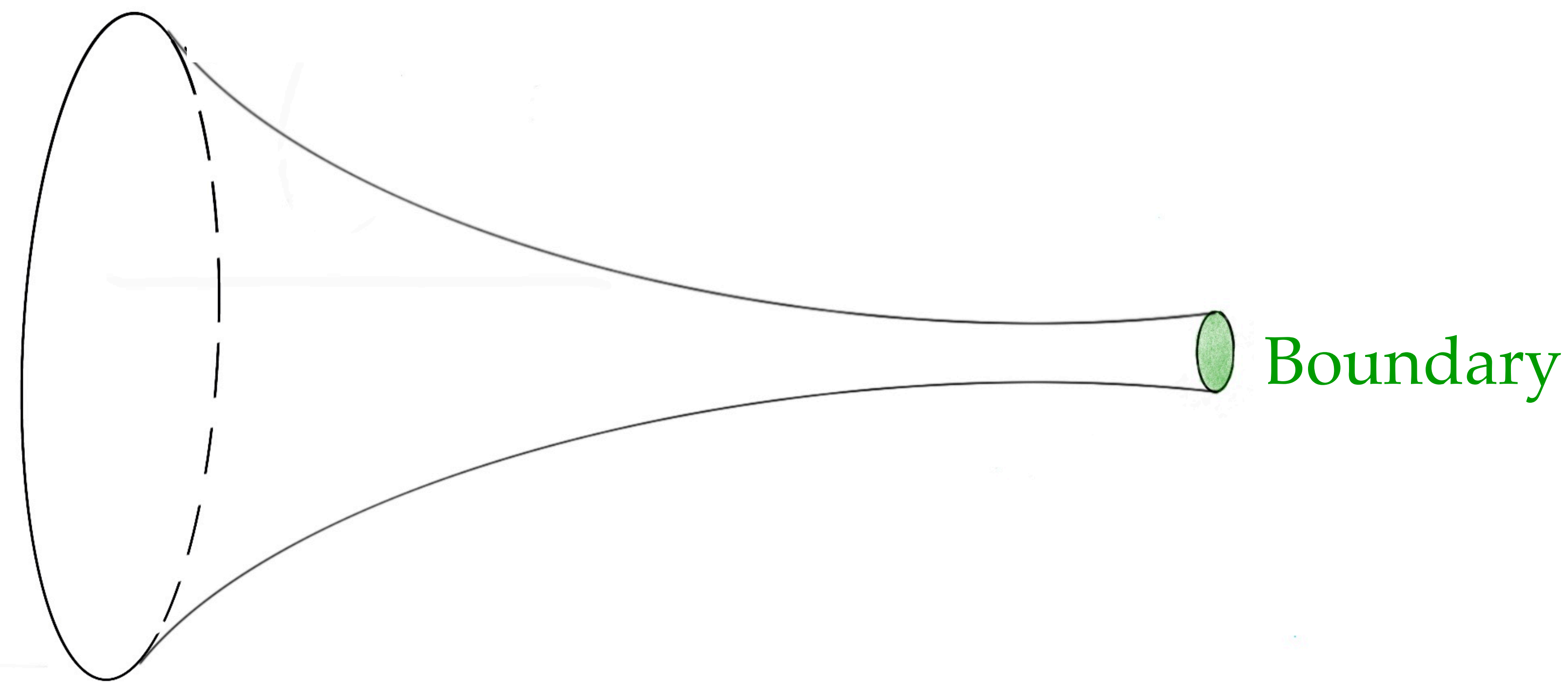

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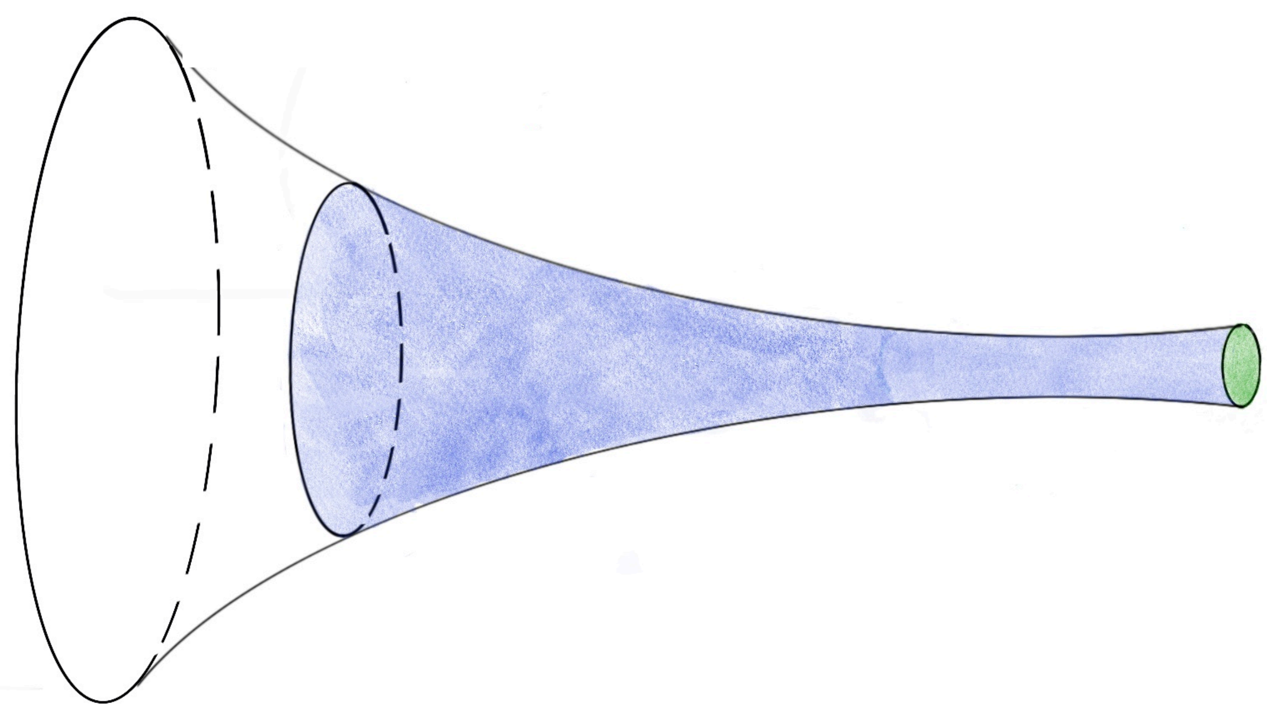

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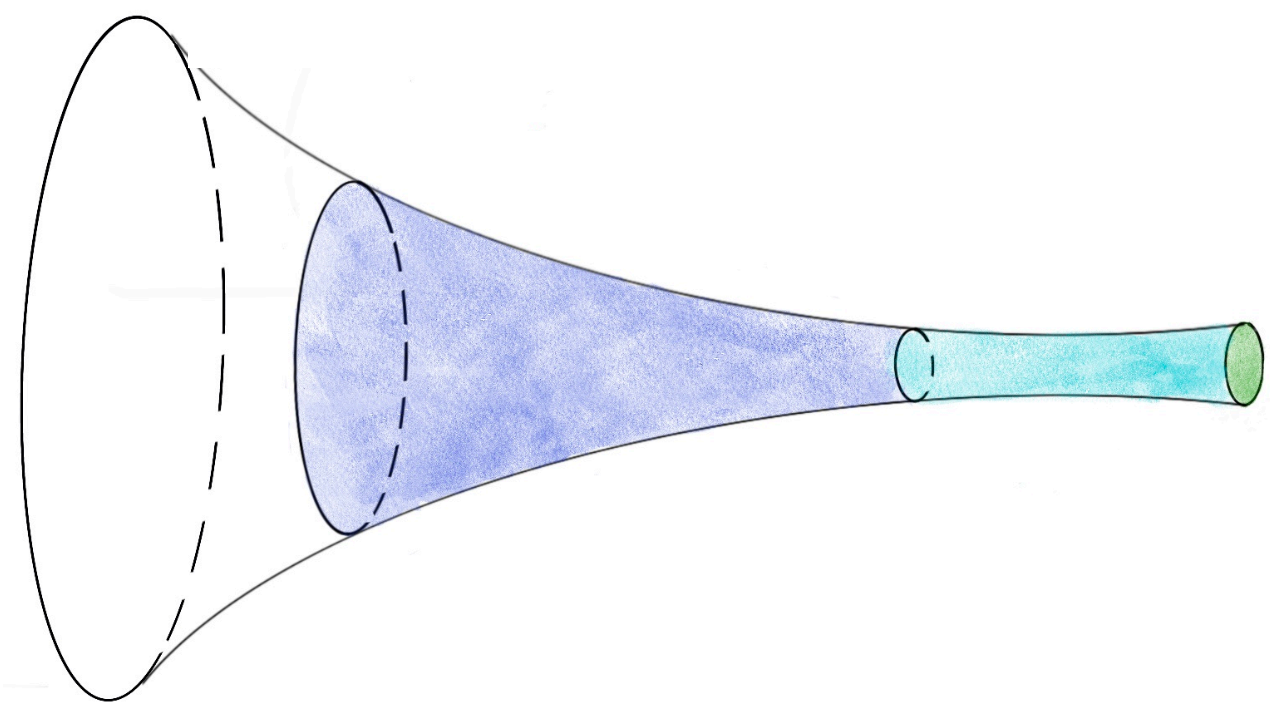

- **Strict** Asymptotic Region ⟶ Introduce an **ordering** (drop polynomial corrections)

$$\frac{s^1}{s^2} > \gamma, \quad \frac{s^2}{s^3} > \gamma, \quad \ldots, \quad \frac{s^{n-1}}{s^n} > \gamma, \quad s^n > \gamma \qquad \gamma \gg 1$$

Asymptotic Hodge Theory
-Boundary structures-

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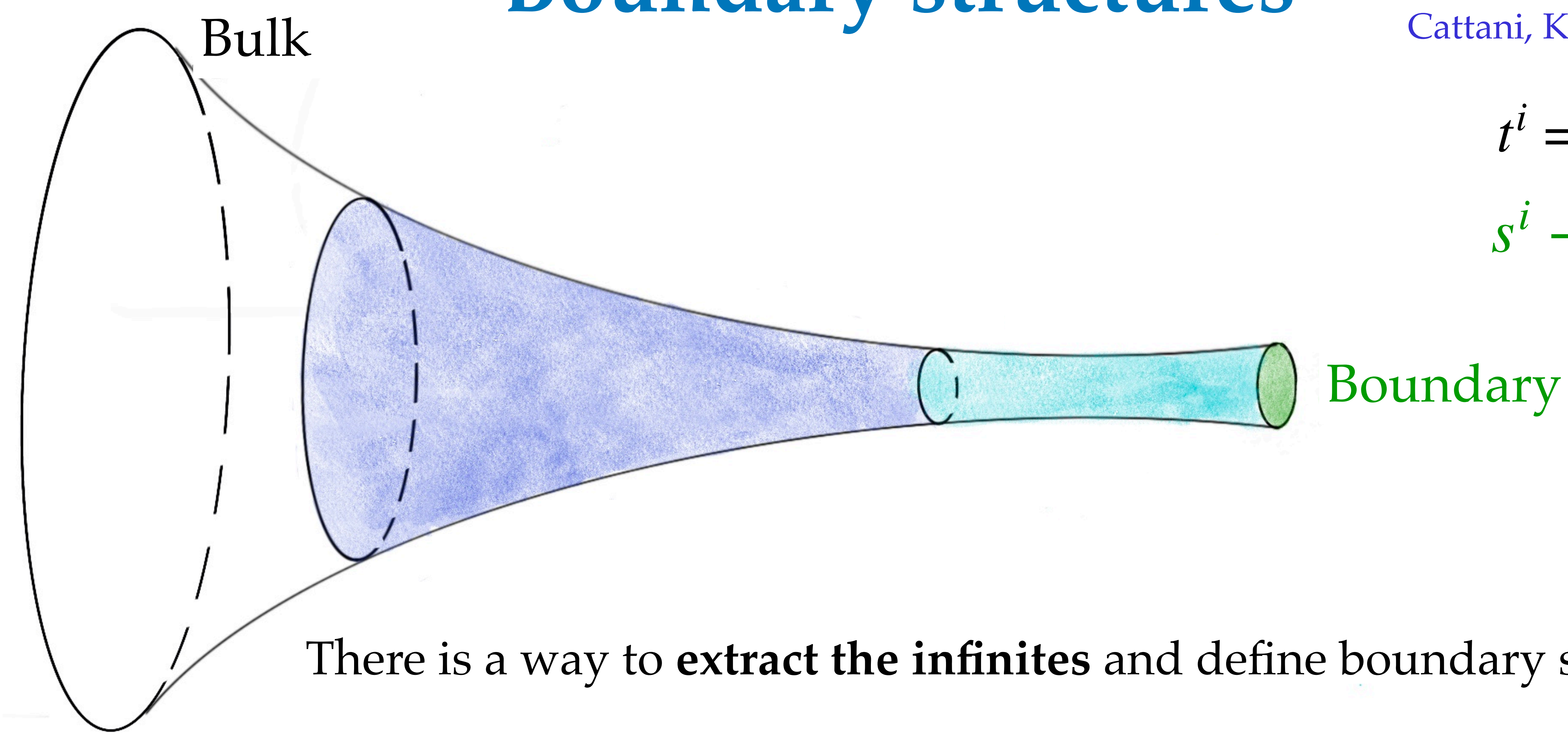

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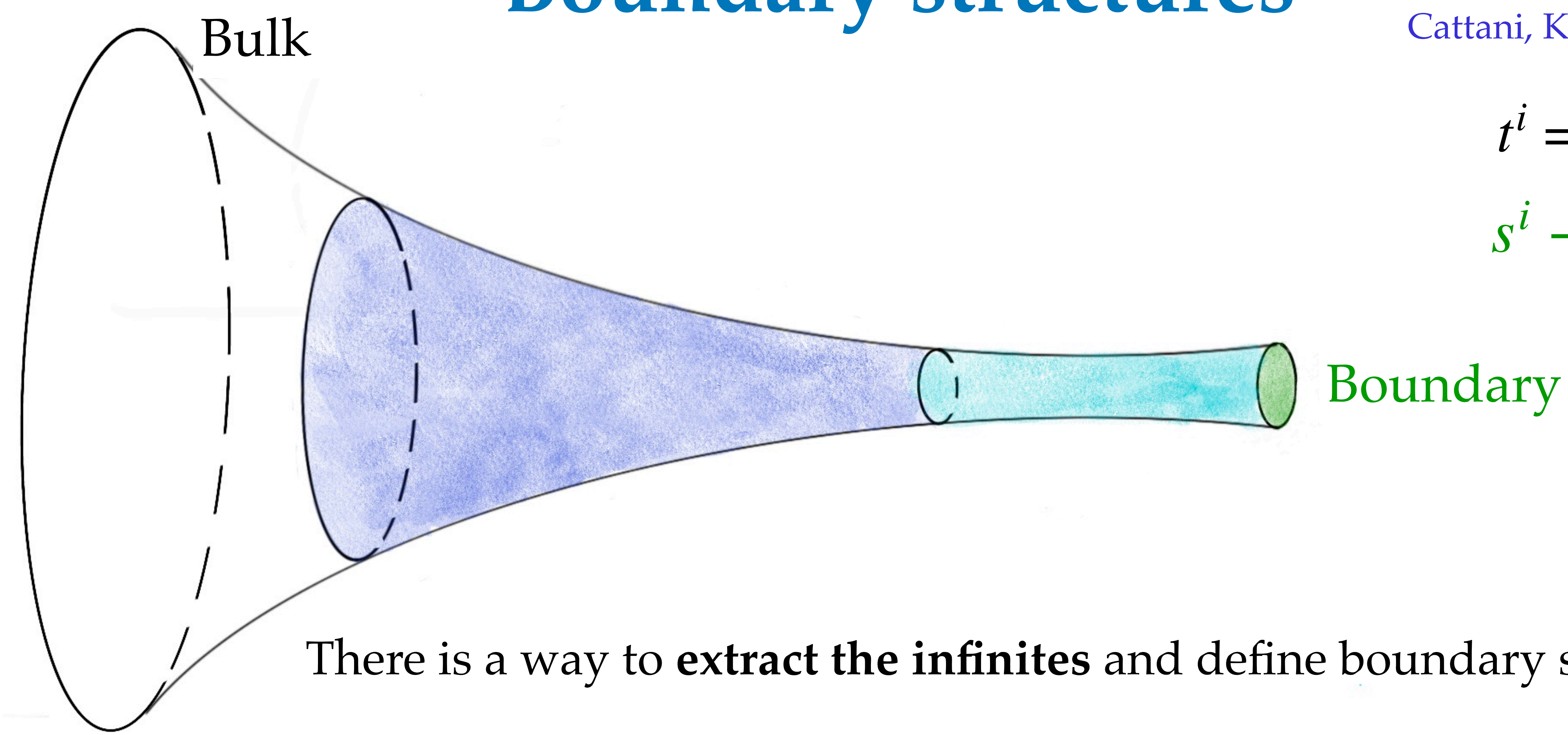

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- Boundary Hodge Decomposition: $H^4_{\text{prim}}(Y_4, \mathbb{C}) = H^{4,0}_\infty \oplus H^{3,1}_\infty \oplus H^{2,2}_\infty \oplus H^{1,3}_\infty \oplus H^{0,4}_\infty$

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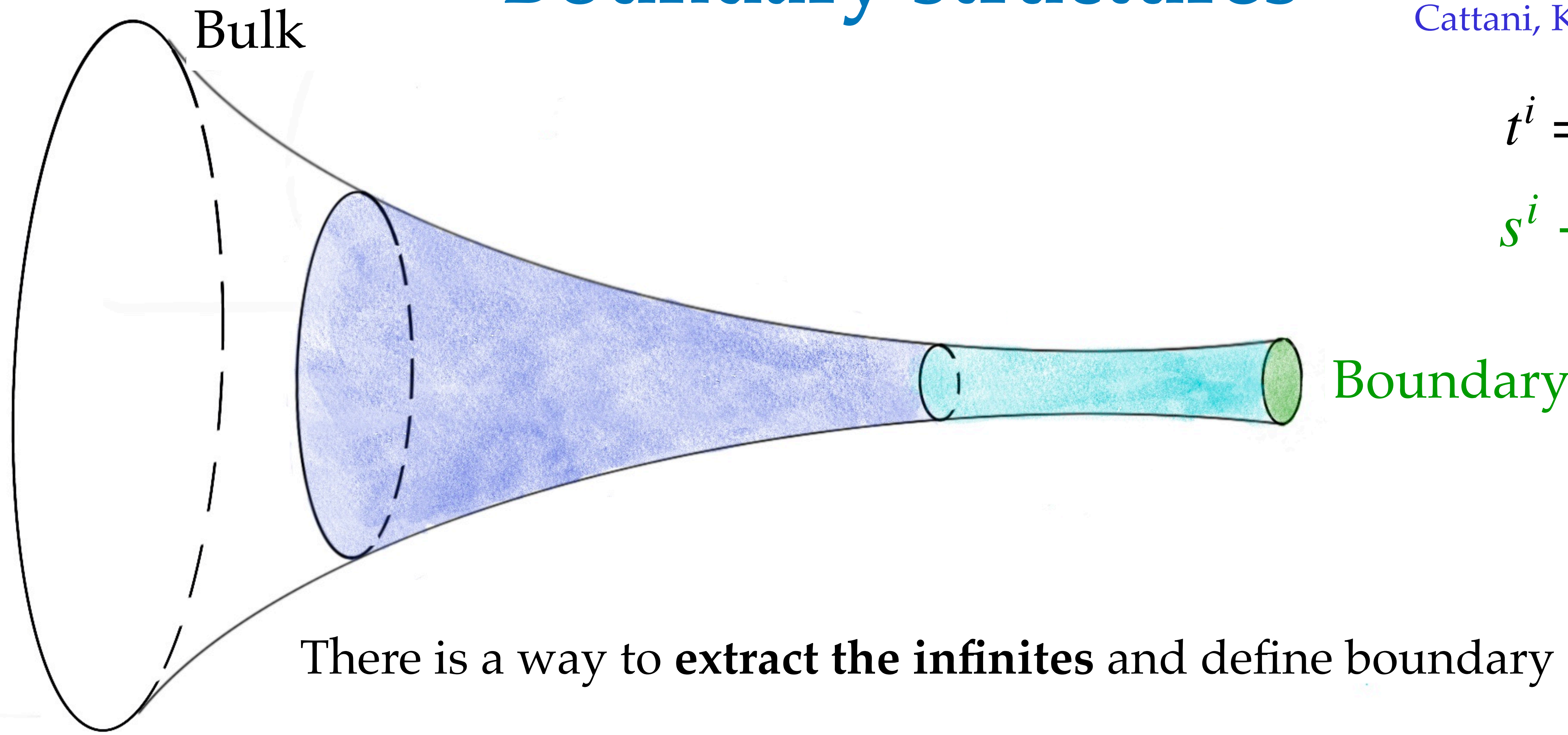

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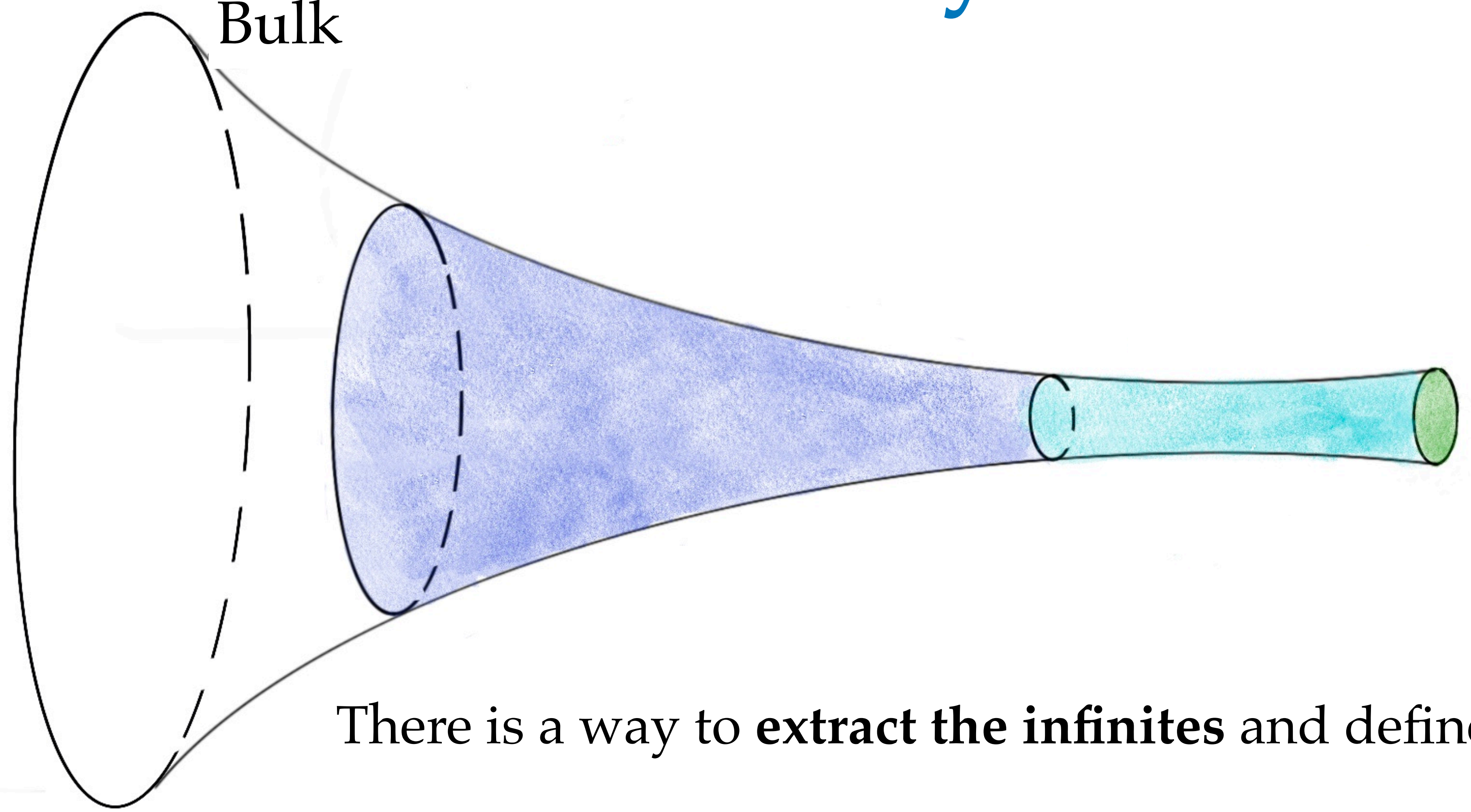

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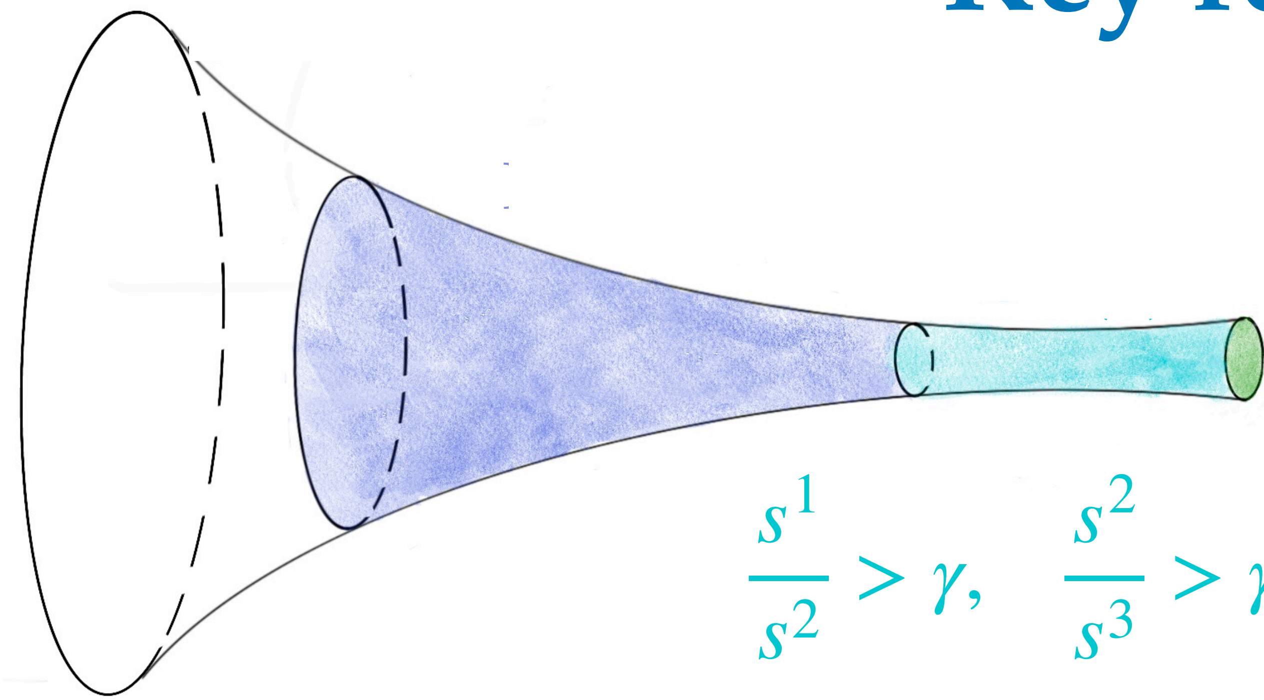
- sl(2) splitting: n **commuting** sl(2) triplets: $\{N_i^-, N_i^+, N_i^0\}$
- Decompose fluxes into sl(2) reps: $H^4_{\text{prim}}\left(Y_4, \mathbb{R}\right) = \bigoplus V_\ell$ $\qquad \ell = (\ell_1, \ldots, \ell_n)$

Asymptotic Hodge Theory
-Key results-



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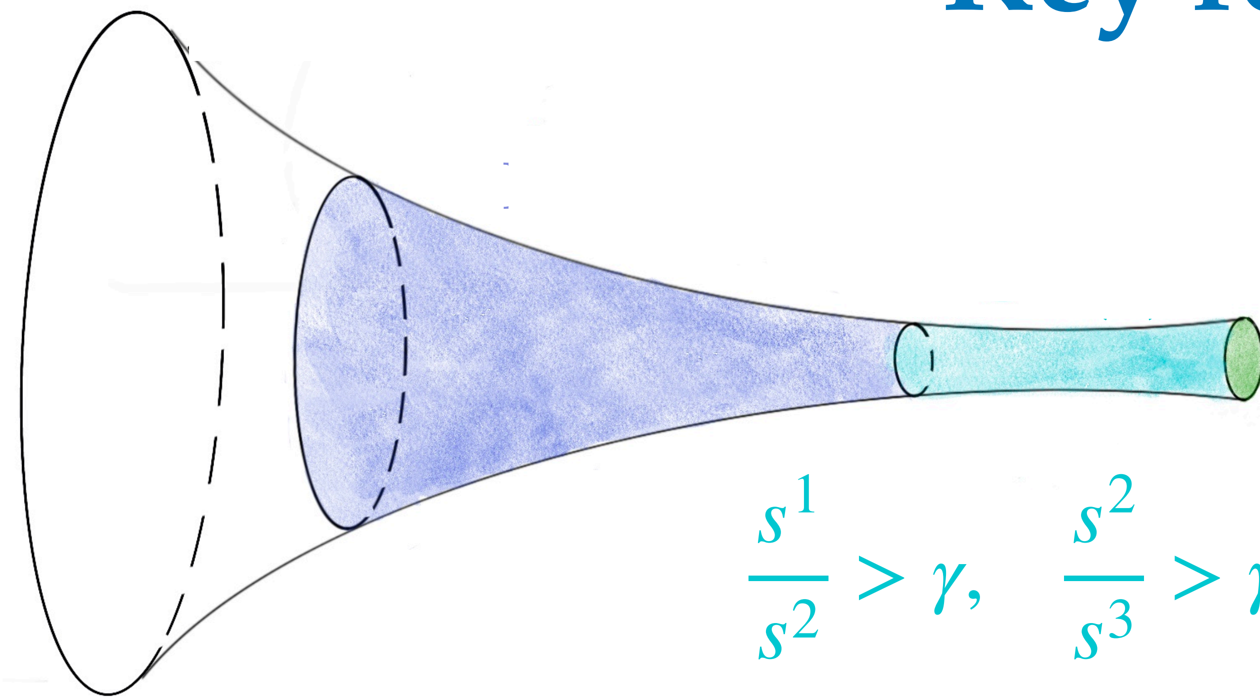

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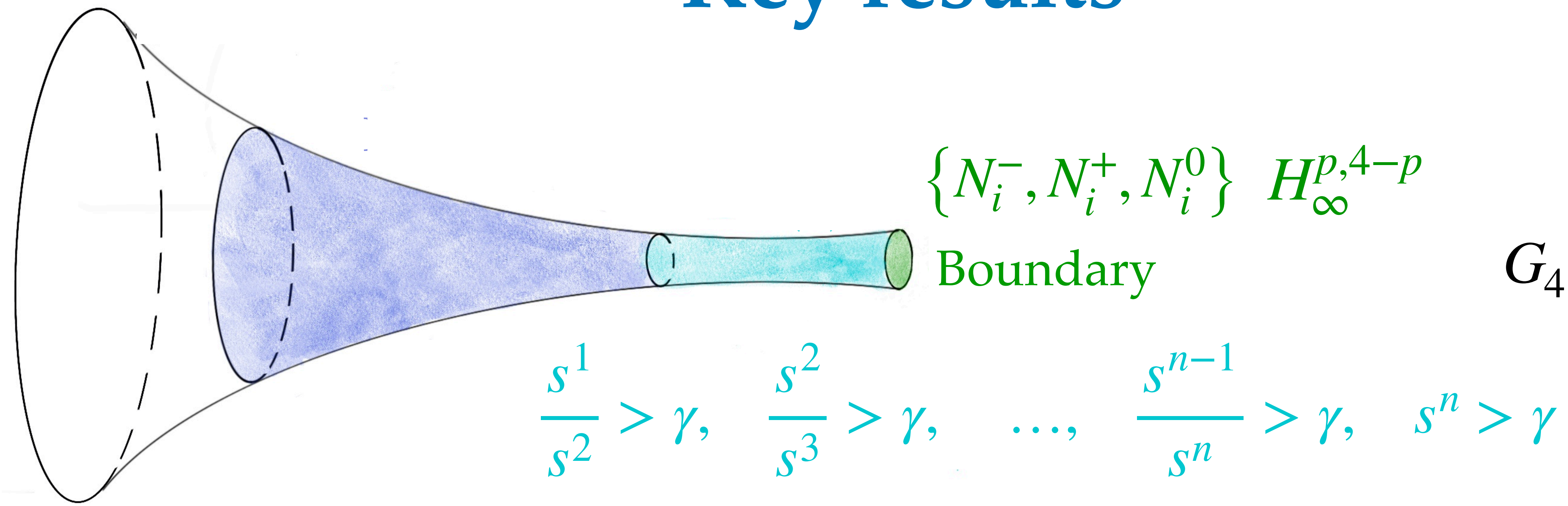

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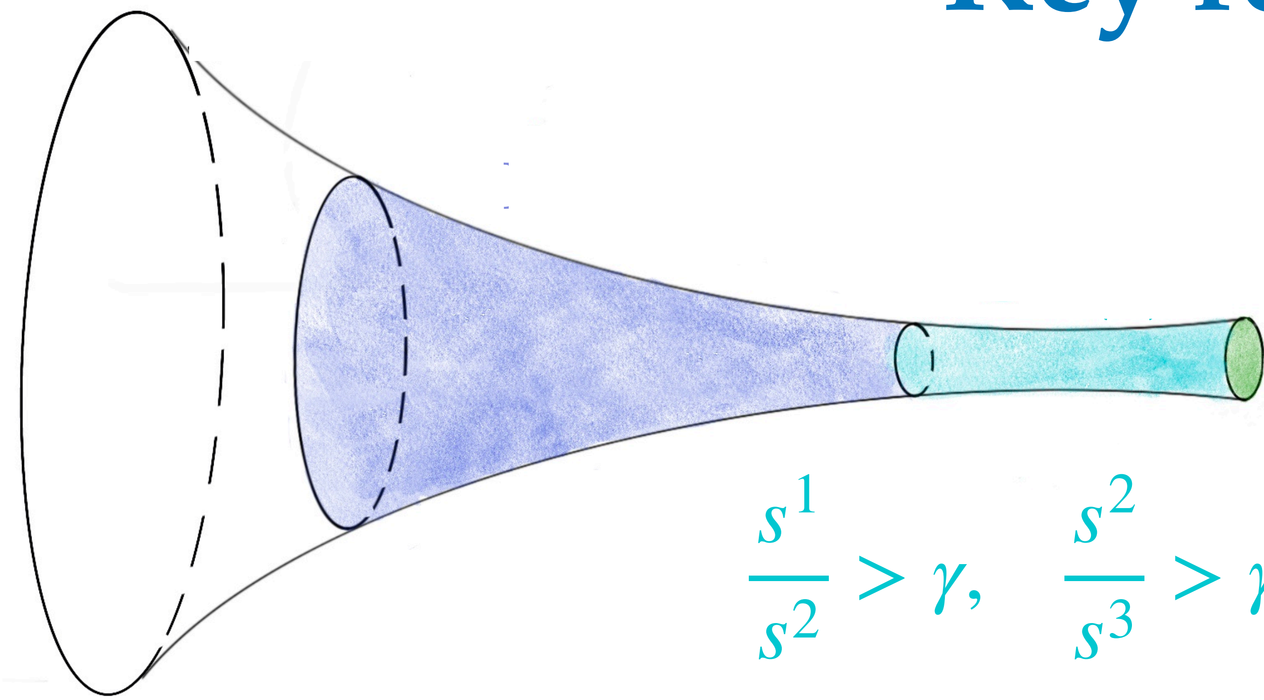

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- Bdry and sl(2) Hodge star: $\star \longrightarrow \star_{\text{sl}(2)}$

$$\begin{cases} \star_\infty : V_\ell \to V_{-\ell} \\ ||v_\ell||^2_{\text{sl}(2)} = \left(\dfrac{s^1}{s^2}\right)^{\ell_1} \left(\dfrac{s^2}{s^3}\right)^{\ell_2} \cdots \left(\dfrac{s^{n-1}}{s^n}\right)^{\ell_{n-1}} (s^n)^{\ell_n} \; ||v_\ell||^2_\infty \end{cases}$$

Moduli stabilization: Generalities

[Grimm, Li, Valenzuela '20]

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- Combining many sl(2) reps only imposes compatibility constraints between the fluxes, but never lowers the tadpole for a fixed modulus.
- Tadpole-wise, the **most economic thing** is to turn on only **one sl(2) per modulus**.

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ℓ' labels the highest weight within each sl(2) rep

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$$\geq 2 \sum_q \left(\frac{s^1}{s^2}\right)^{\ell'_1} \cdots \left(s^n\right)^{\ell'_n} \; ||G_{q,\ell'}||^2_\infty$$

$\downarrow$

n positive-definite terms (at least)

q labels sl(2)-representations

$\ell > 0$ labels positive weights

ℓ' labels the highest weight within each sl(2) rep

Tadpole Contribution

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↓

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At least linear scaling with (a large number of) stabilized moduli

Moduli stabilization and Tadpole -Key results-

- For **large number of stabilized moduli**, most of them (i.e. at least $(n-4)$ of them) have to be **fixed with fluxes from these representations**

$$P_0 \qquad P_{01_i} \qquad P_{02_i} \qquad P_{01_i2_j}$$

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- Key **qualitative difference** between **small and large number of moduli**

Outlook & Open Questions

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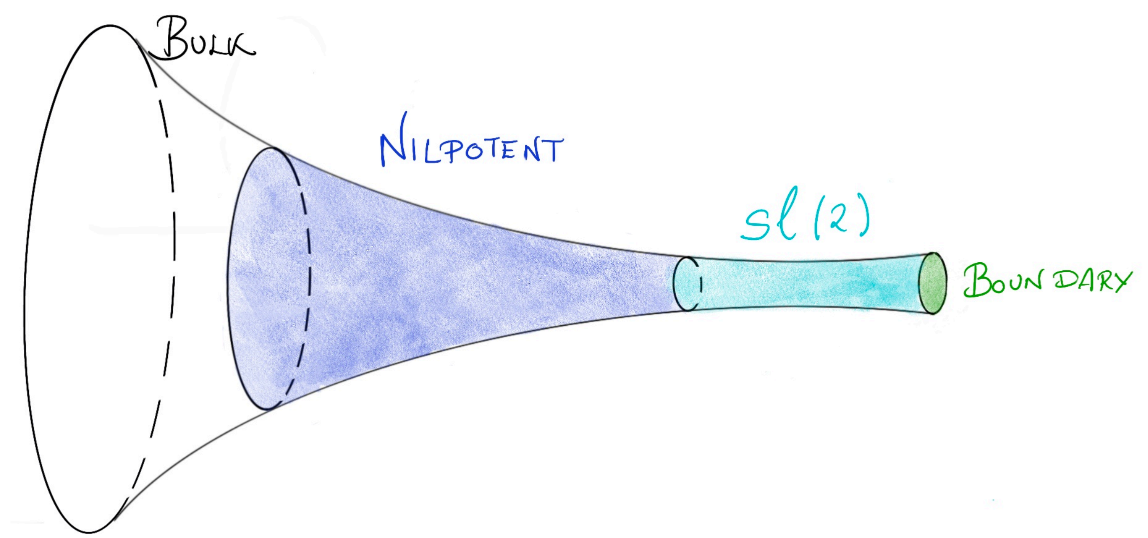

- Extend to the **interior of moduli space**

  1) Include polynomial corrections (~~strict~~ asymptotic limit) $\longrightarrow$ "Linear scenario"

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  [Marchesano, Prieto, Wiesner '21]
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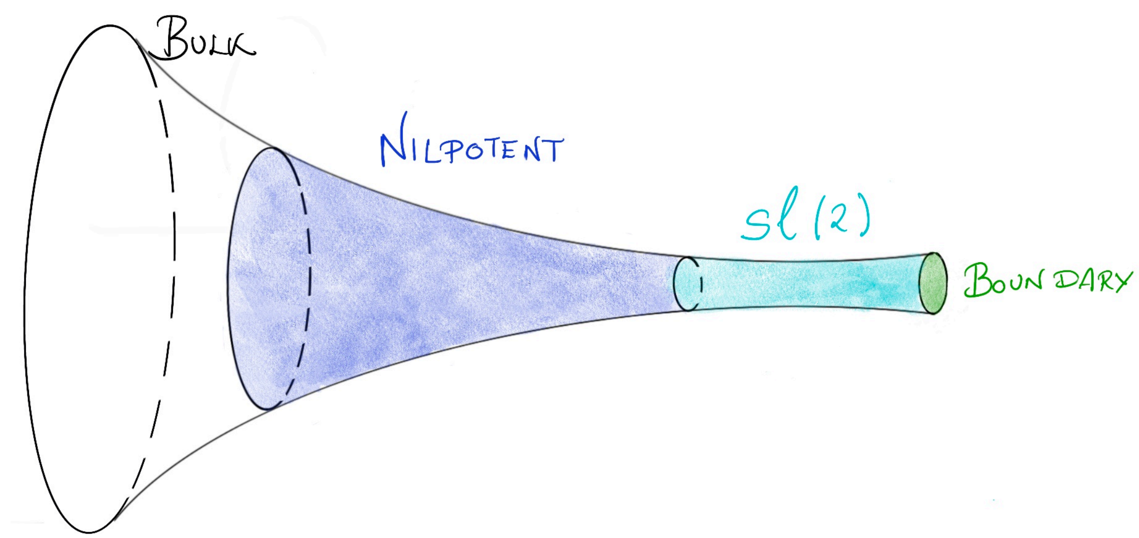

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شكراً جزيلاً
ngiyabonga
рахмат
danke
謝謝
teşekkür ederim
Баярлалаа
mersi
kia ora
barka
welalin
tack
spas
mahalo
tapadh leat
спасибо
faafetai lava
vinaka
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blagodaram
dank je
misaotra
matondo
paldies
grazzi
kiitos
dankie
nanni
nandri
dhanyavad
thank you
gracias
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asante
manana
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dhanyavadagalu
shukriya
mercê
мерси
তোমাকে ধন্যবাদ
감사합니다
xièxie
merci

Backup slides

Charge quantization and extensions

- **Charge quantization** in the sl(2)-basis (over $\mathbb{Q}$ instead of over $\mathbb{Z}$) ⟶

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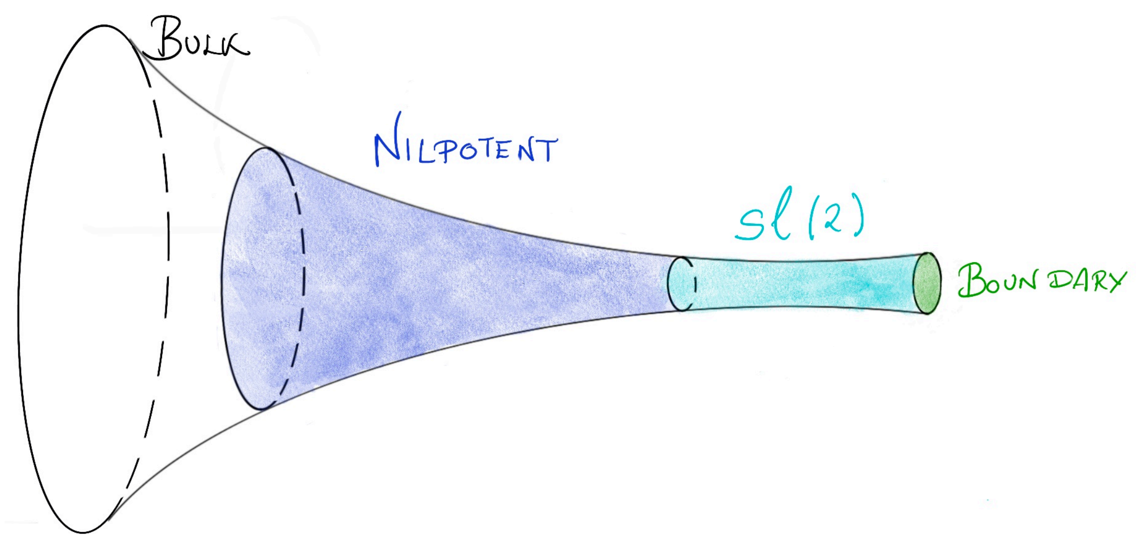

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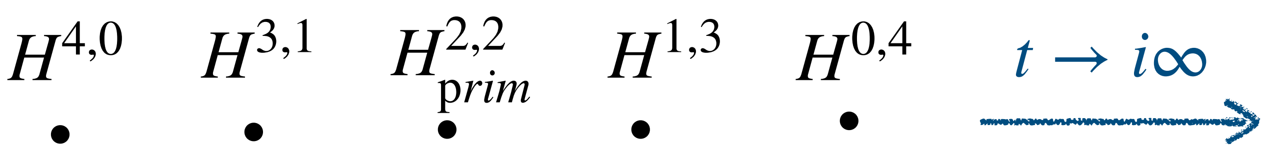
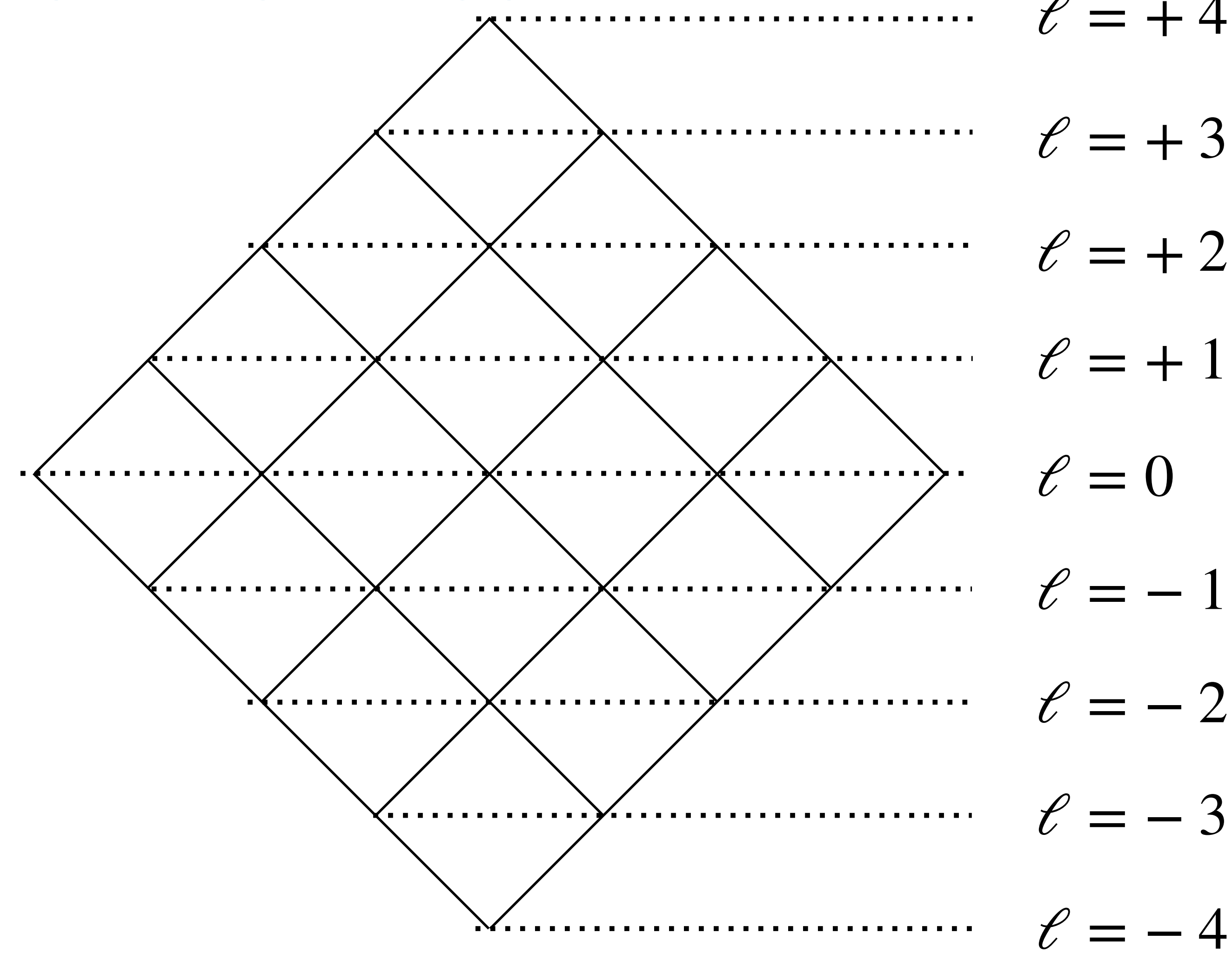
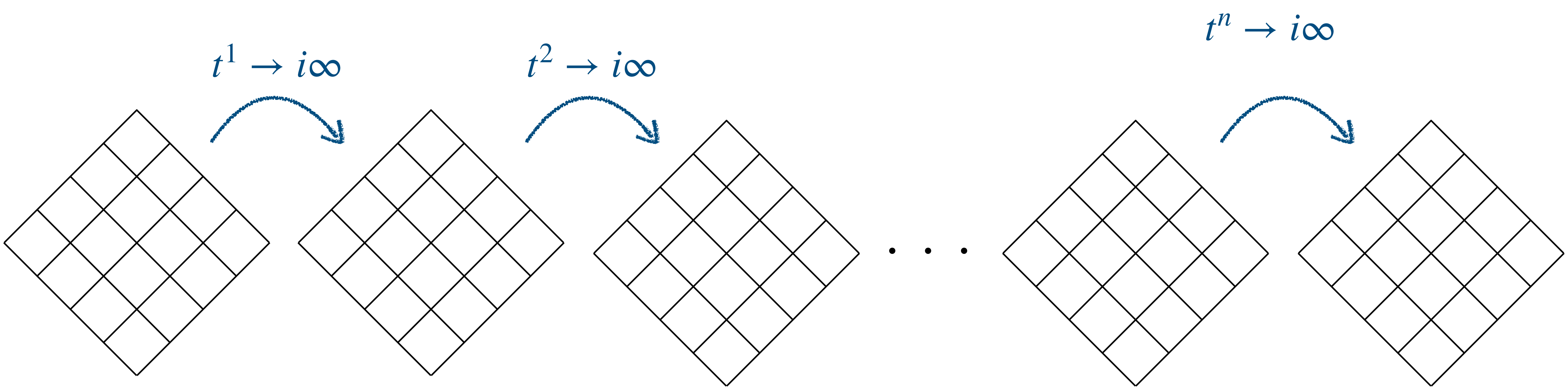
Asymptotic Hodge Theory
-Boundary structures-



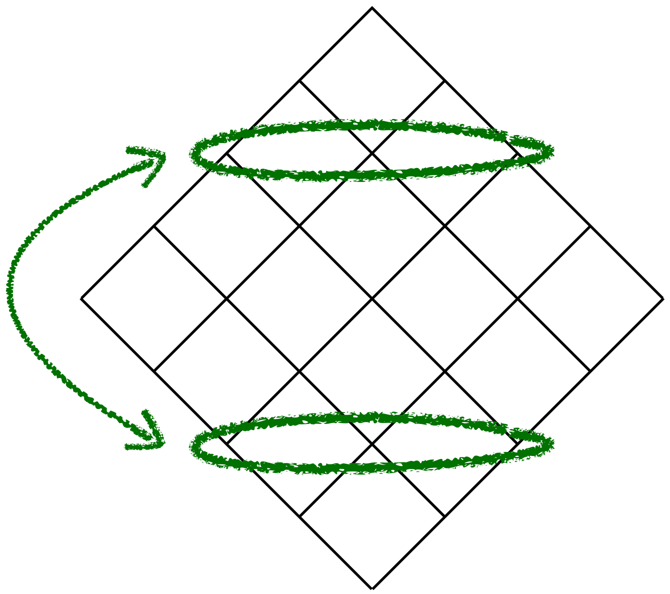


$$H^4_{\text{prim}}\left(Y_4, \mathbb{R}\right) = \bigoplus V_\ell$$



Asymptotic Hodge Theory
-Hodge star close to the boundary-

- The boundary Hodge decomposition naturally includes a boundary Hodge star operator:



$$\star_\infty : V_\ell \to V_{-\ell}$$

$$\langle v_\ell, v_{\ell'} \rangle = 0 \qquad \text{for } \ell \neq -\ell'$$

$$\langle v_\ell, \star_\infty v_{\ell'} \rangle = 0 \qquad \text{for } \ell \neq \ell'$$

- Allows to express the Hodge star in the strict asymptotic limit: $\star \longrightarrow \star_{\mathrm{sl}(2)}$

Vanishing axions: $\star_{\mathrm{sl}(2)}\, v_\ell = \left(\frac{s^1}{s^2}\right)^{\ell_1} \left(\frac{s^2}{s^3}\right)^{\ell_2} \cdots \left(\frac{s^{n-1}}{s^n}\right)^{\ell_{n-1}} (s^n)^{\ell_n} \star_\infty v_l$

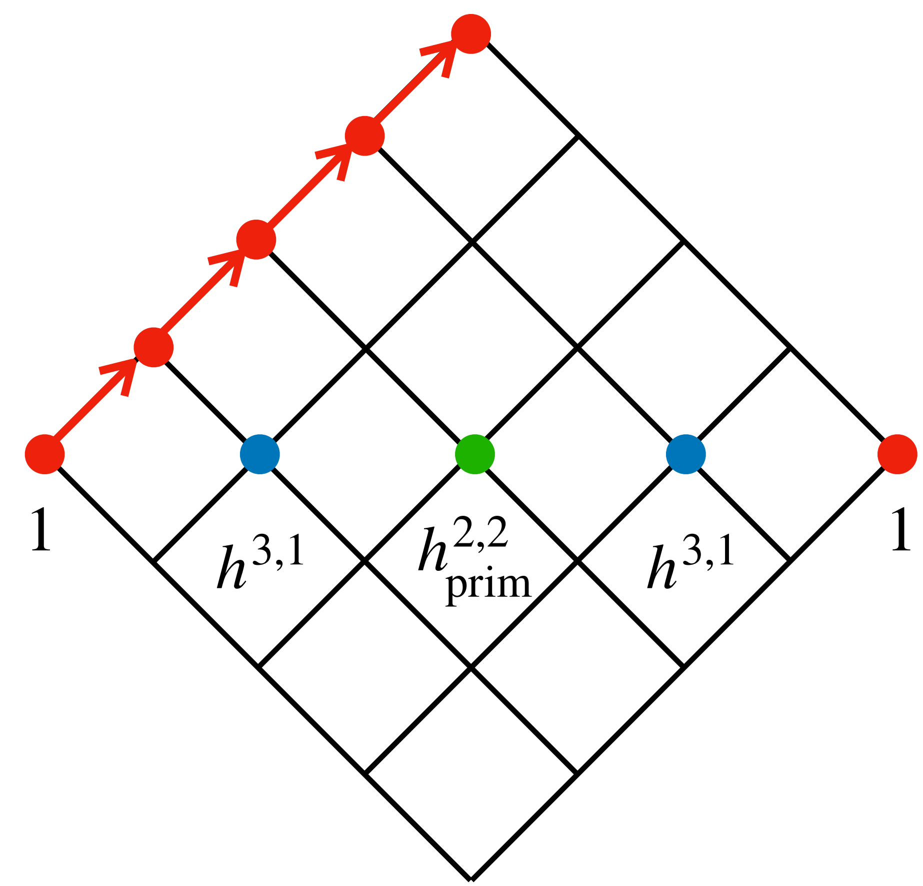
Non-vanishing axions ⟶ Mixing with lower subspaces via $e^{\phi^i N_i^-}\, v_l$

Asymptotic Hodge Theory -Highest weight spaces-

- Elements in $H^4_{\text{prim}}(Y_4, \mathbb{R})$ arrange into irreps of the n commuting sl(2) ⟶ Generated by applying N_i^- to **highest weight states**

- What are the possible highest weight states?

Only **ONE**, corresponding to the **(4,0)-form**, can move along the exterior line of the diagram



Asymptotic Hodge Theory
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- Elements in $H^4_{\text{prim}}(Y_4, \mathbb{R})$ arrange into irreps of the n commuting sl(2) $\longrightarrow$ Generated by applying N_i^- to **highest weight states**

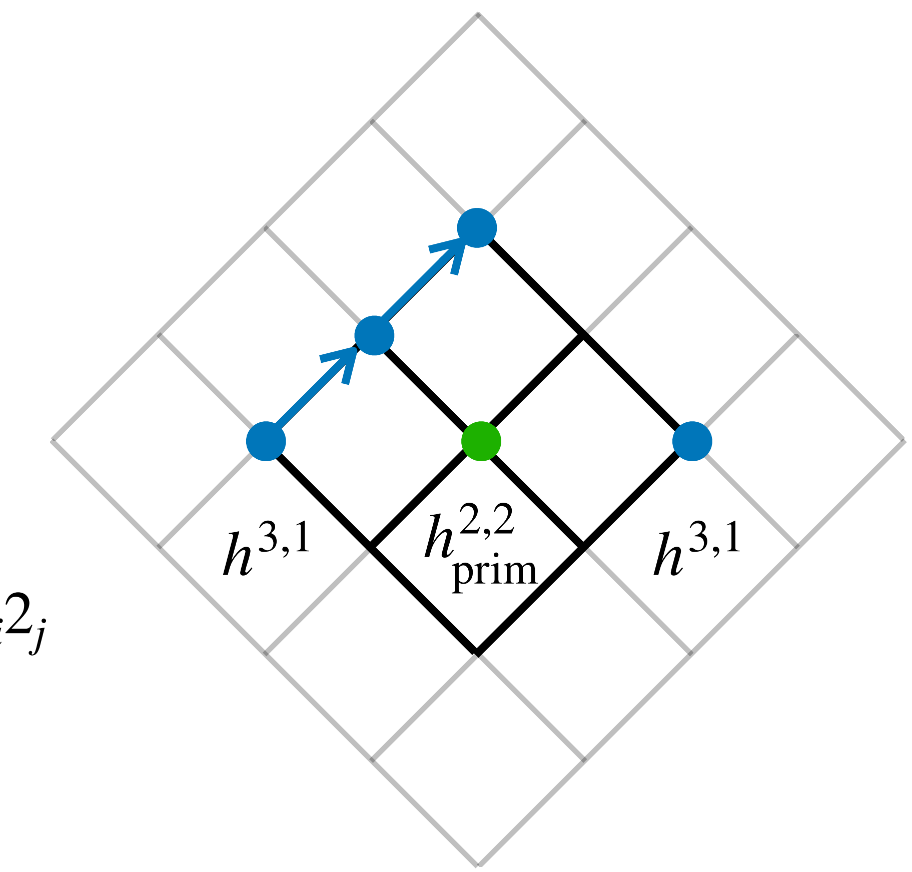
- What are the possible highest weight states?

P_0

$P_{01_i} \to N_i^- P_{01_i}$

$P_{02_i} \to N_i^- P_{02_i} \to (N_i^-)^2 P_{02_i}$

$P_{01_i 2_j} \to N_j^- P_{01_i 2_j}$, $P_{01_i 2_j} \to N_i^- P_{01_i 2_j}$, $N_j^- P_{01_i 2_j} \to N_i^- N_j^- P_{01_i 2_i}$, $N_i^- P_{01_i 2_j} \to N_i^- N_j^- P_{01_i 2_i}$



~ $h^{3,1}$ copies of K3 surfaces

Moduli stabilization at work

$$\hat{G}_{-\ell} = \left(\frac{s^1}{s^2}\right)^{\ell_1} \cdots \left(\frac{s^{n-1}}{s^n}\right)^{\ell_{n-1}} \left(s^n\right)^{\ell_n} \star_\infty \hat{G}_{+\ell}$$

$$\hat{G}_4 = e^{\phi^i N_i^-} G_4$$

- Example: Flux from the sl(2) rep generated by P_{02_i}

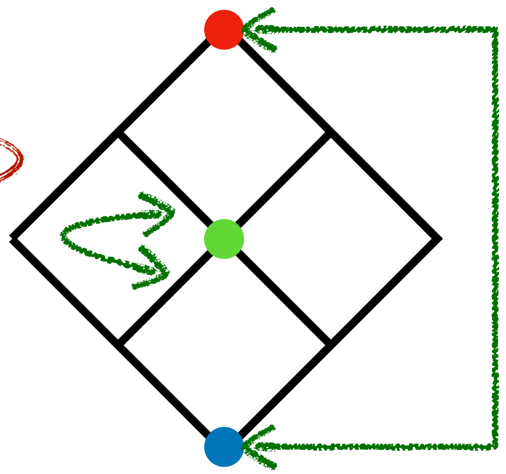
$$G_4 = G_{02}\, v_{02} + G_0\, v_0 + G_{0-2} v_{0-2} \longrightarrow Q = \frac{1}{2}\langle G_4, G_4\rangle = G_{02}G_{0-2} - \frac{1}{2}G_0^2$$

$$\hat{G}_4 = G_{02}\, v_{02} + \left(G_0 - \phi\, G_{02}\right) v_0 + \left(G_{0-2} - \phi\, G_0 + \frac{1}{2}(\phi)^2 G_{02_i}\right) v_{0-2}$$

- Self-duality conditions:

$$\frac{G_{02}}{2}(s)^2 = G_{0-2} - \phi G_0 + \frac{1}{2}(\phi)^2 G_{02} \longrightarrow s = \frac{\sqrt{2G_{0-2}G_{02} - G_0^2}}{G_{02}}$$

$$\left(G_0 - \phi G_{02}\right) = -\left(G_0 - \phi G_{02}\right) \longrightarrow \phi = \frac{G_0}{G_{02}}$$



sl(2) Hodge star

- Extend boundary Hodge star to the interior: $\star_{\infty} \longrightarrow \star_{\mathrm{sl}(2)}$

$$\star_{\mathrm{sl}(2)} = e^{+\phi^i N_i^-} \left[e^{-\frac{1}{2}\log(s^i) N_i^0} \star_{\infty} e^{+\frac{1}{2}\log(s^i) N_i^0} \right] e^{-\phi^i N_i^-}$$

$$H^4_{\mathrm{prim}}(Y_4, \mathbb{C}) = H^{4,0}_{\mathrm{sl}(2)} \oplus H^{3,1}_{\mathrm{sl}(2)} \oplus H^{2,2}_{\mathrm{sl}(2)} \oplus H^{1,3}_{\mathrm{sl}(2)} \oplus H^{0,4}_{\mathrm{sl}(2)}$$

$$H^{p,q}_{\mathrm{sl}(2)} = e^{\phi^i N_i^-} e^{-\frac{1}{2}\log(s^i) N_i^0} H^{p,q}_{\infty}$$